Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name: Discrete Mathematics

Subject Code: 4SC05DMC1 **Branch: B.Sc.(Mathematics)**

Semester: 5 **Date:31/03/2018** **Time :10:30 To 01:30** **Marks : 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Define the terms:** **(14)**

a) Equivalence Relation. **(01)**

b) Partially order Relation. **(01)**

c) Chain. **(01)**

d) Greatest lower bound of set. **(01)**
e) Maximal element. **(01)**
f) Lattice. **(01)**
g) Lattice as an algebraic system. **(01)**
h) Sub lattice. **(01)**
i) Complete lattice. **(01)**
j) Bounded lattice. **(01)**
k) Atom. **(01)**
l) Join – irreducible element. **(01)**
m) Boolean homomorphism. **(01)**
n) Fuzzy subset. **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Show that relation $R = \{(a,b) | \; a \equiv b \; (mod \; m)\}$ is a equivalent relation on the set of integers. **(05)**

b) Show that $< N, D >$ is a partial order set, where Ddivides relation. **(05)**

c) Draw the Hass diagram of $< S_{60}, D >$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) If $< A, \leq_1 >$ and $< B, \leq_2 >$ be two Posets then prove that $< A \times B, \leq >$ is Poset, **(06)**

where relation $\leq$ defines by $(a, b) \leq (c, d)$ if $a \leq_1 c$ and$b \leq_2 d$ for $\forall\ (a, b), (c, d) \in A \times B$.

b) Prove that $< S_{30}, D >$ is a lattice. **(06)**

c) Prove that least upper bound ofsubset of a Partially order set is unique. **(02)**

Q-4 **Attempt all questions** **(14)**

a) Let $< L, \leq >$ be a lattice and $a, b, c \in L$then prove the following **(06)**

(i) $a * b = b * a$

(ii) $a * (a \oplus b) = a$.

(iii) If $b \leq c$ then prove that $a * b \leq a * c, \forall\ a \in L$

b) State and prove distributive inequality. **(06)**

c) Let $< L, \leq >$ be a lattice and $a, b, c \in L$. If $a \leq b \leq c$ then prove that **(02)**

$$a \oplus b = a * b$$

Q-5 **Attempt all questions** **(14)**

a) Obtain SOP canonical form of $\alpha(x_1, x_2, x_3) = x_1 \oplus (x_2 * x_3')$ in three variables. **(07)**

b) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 * (x_2 \oplus x_3)$ in a cube array method. **(05)**

c) Let $< L, *, \oplus, ', 0, 1 >$is complemented distributive lattice. If $a, b \in L$ with$a \leq b$then prove that $a * b' = 0$. **(02)**

Q-6 State and prove Stone's representation theorem. **(14)**

Q-7 **Attempt all questions** **(14)**

a) Let $(L, *, \oplus, ', 0, 1)$ is complemented distributive lattice. If $a, b, c \in L$ then prove the following **(06)**

(i) $(a * b)' = a' \oplus b'$.

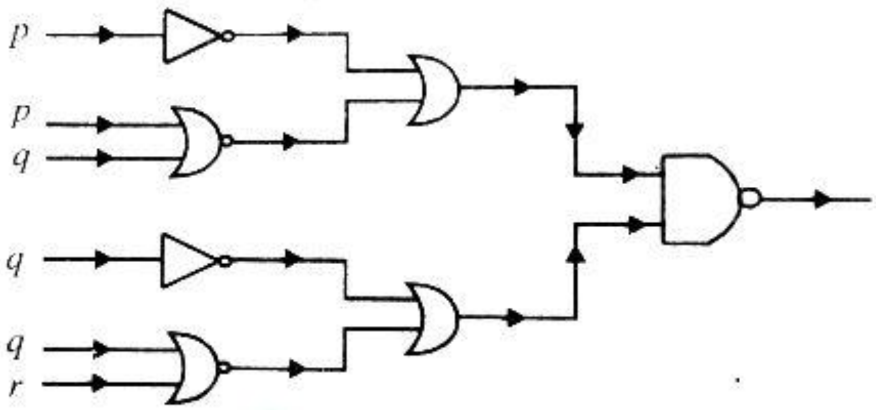
(ii) $(a \oplus b)' = a' * b'$.

b) If a, b are two different atoms of a Boolean algebra then prove that $a * b = 0$. **(04)**

c) Let $< B, *, \oplus, 0, 1)$ be a Boolean algebra. Then prove that non – element $a \in B$ is an atom of Bif and only if $a * x = 0$ or $a * x = a \forall\ x \in B$. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Simplify the circuit given in following figure using Boolean identities. **(07)**



b) Let $E=\{a,b,c,d,e\}$, $\underset{\sim}{A}=\{(a,0.3),(b,0.8),(c,0.5),(d,0.1),(e,0.9)\}$, **(07)**

$\underset{\sim}{B}=\{(a,0.7),(b,0.6),(c,0.4),(d,0.2),(e,0.1)\}$ then find the following:

(i) $\underset{\sim}{B}'$ (ii) $\underset{\sim}{A} \cdot \underset{\sim}{B}$ (iii) $\underset{\sim}{A} \overset{\wedge}{+} \underset{\sim}{B}$ (iv) $\underset{\sim}{A} - \underset{\sim}{B}$ (v) $\underset{\sim}{A} \cap \underset{\sim}{B}$ (vi) $\left(\underset{\sim}{A}'\right)'$ (vii) $\underset{\sim}{A} \cup \underset{\sim}{B}$